$r = 2\cos(3\theta)$

parabola

Steps
①
②

5) $3x^2 - 2y^2 - 6x - 12y - 27 = 0$

$3x^2 - 2y^2 - 6x - 12y = 27$

$(3x^2 - 6x \quad)$

$3(x^2 - 2x + 1) - 2(y + 6y + 9) = 27 + 3 + (-18)$

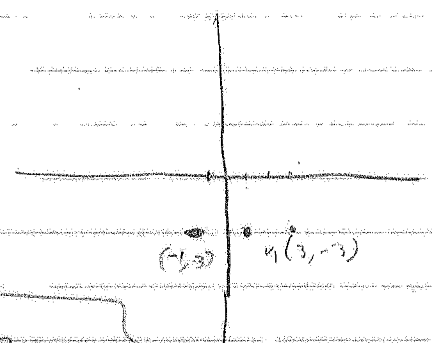
$\left[\frac{1}{2}(-2)\right]^2 = 1 \qquad \left(\frac{1}{2}(6)\right)^2 = 9$

$3(x-1)^2 - 2(y+3)^2 = 12$

$\frac{(x-1)^2}{4} - \frac{(y+3)^2}{6} = 1$

center $(1, -3)$ $\quad a^2 = 4,\ a = 2$

$b^2 = 6;\ b = \sqrt{6}$



13) $x(t) = 2t - t^2$ $\qquad$ from $1 \le t \le 2$

$y(t) = 2t^{3/2}$

$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\frac{d}{dt}(x) = \frac{d}{dt}\left[2t - t^2\right]$ $\qquad$ $\frac{d}{dt}[y] = \frac{d}{dt}\left[2t^{\frac{3}{2}}\right]$

$\frac{dx}{dt} = 2 - 2t$ $\qquad$ $\frac{dy}{dt} = \frac{3}{2} \cdot 2t^{\frac{1}{2}}$

$\frac{dy}{dt} = 3t^{\frac{1}{2}}$

$(2-2t)(2-2t)$

$4 - 4t - 4t + 4t^2$

$4 - 8t + 4t^2$

Length $= \int_1^2 \sqrt{(3t^{\frac{1}{2}})^2 + (2-2t)^2}\, dt$

$= \int_1^2 \sqrt{9t + (4 - 8t + 4t^2)}\, dt$

$= \int_1^2 \sqrt{t + 4 + 4t^2}\, dt$

$= \int_1^2 \sqrt{4t^2 + t + 4}\, dt$

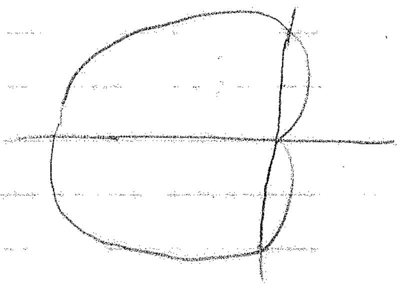
function, X, lower Bound, upper bound

#.

#20 find arc length . the interval $0 \leq \theta \leq 2\pi$

$f(\theta) = 2 - 2\cos\theta$

$f'(\theta) = 2\sin\theta$



$$A = 2\int_0^{\pi} \sqrt{(f(\theta))^2 + (f'(\theta))^2}$$

$$= 2\int_0^{\pi} \sqrt{(2-2\cos\theta)^2 + (2\sin\theta)^2}$$

$$= 2\int_0^{\pi} \sqrt{(4 + 4\cos^2\theta - 8\cos\theta) + (4\sin^2\theta)}$$

$$= 2\int_0^{\pi} \sqrt{4\cos^2\theta + 4\sin^2\theta + 4 - 8\cos\theta}$$

$$= 2\int_0^{\pi} \sqrt{4 + 4 - 8\cos\theta}$$

$$= 2\int_0^{\pi} \sqrt{8 - 8\cos\theta}$$

$$= 2\int_0^{\pi} \sqrt{8(1-\cos\theta}$$

$$= 2\sqrt{8}\int_0^{\pi} \sqrt{1-\cos\theta} \qquad \frac{1-\cos x}{2} = \sin^2\left(\frac{x}{2}\right)$$

$$= 2\sqrt{8}\int_0^{\pi} \sqrt{2\sin^2\tfrac{\theta}{2}} \qquad 1 - \cos x = 2\sin^2\tfrac{x}{2}$$

$$= 2\sqrt{8}\cdot\sqrt{2}\int_0^{\pi} \sqrt{\sin^2\tfrac{\theta}{2}} \qquad = -8\cdot 8\left[\cos\tfrac{\theta}{2}\right]_0^{\pi}$$

$$= \left[-16\cos\tfrac{\theta}{2}\right]_0^{\pi}$$

$$= 2\sqrt{8}\cdot\sqrt{2}\int_0^{\pi} \sin\left(\tfrac{\theta}{2}\right) \qquad = -16\cos\tfrac{\pi}{2} - (-16\cos 0)$$

$$= 2\sqrt{16}\left[\left(-\cos\tfrac{\theta}{2}\right)(2)\right] \qquad = -16\cdot 0 - (-16(1)) = \boxed{16}$$

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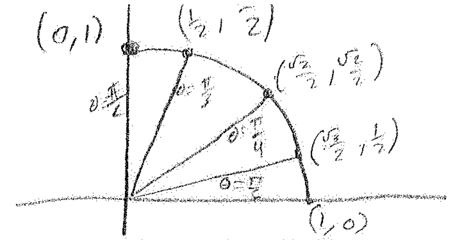


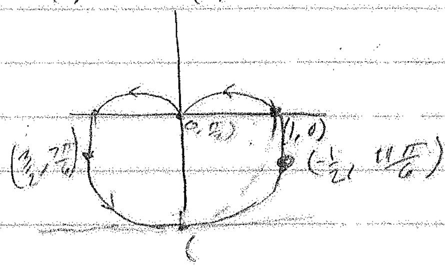
$\boxed{10.4}$ #69

$r(\theta) = 1 - \sin(\theta)$

$r'(\theta) = -\cos(\theta)$

$\frac{dy}{dx}$ = slope of tan. line at a point

$$= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r(\theta)\cdot\cos\theta + r'(\theta)\cdot\sin\theta}{-r(\theta)\cdot\sin\theta + r'(\theta)\cdot\cos\theta}$$



Horizontal tan

when $\frac{dy}{d\theta} = 0$, $\frac{dx}{d\theta} \neq 0$

$\frac{dy}{d\theta} = \cos(\theta)\,[1 - 2\sin\theta] = 0$

$\cos\theta = 0$ and $1 - 2\sin\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad\quad$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$1 - 2\sin\frac{\pi}{6} = 0$

$1 - 2\sin(\frac{5\pi}{6}) = 0$

Vertical tangency

when $\frac{dx}{d\theta} = 0$ and $\frac{dy}{d\theta} \neq 0$

$\frac{dx}{d\theta} = -(1-\sin\theta)\cdot\sin\theta + (-\cos\theta)\cdot\cos\theta$

$= (-1+\sin\theta)(\sin\theta) + (-\cos^2\theta)$

$= -\sin\theta + \sin^2\theta - \cos^2\theta$

$= -\sin\theta + \sin^2\theta - (1 - \sin^2\theta)$

$= -\sin\theta + \sin^2\theta - 1 + \sin^2\theta$

$= 2\sin^2\theta - \sin\theta - 1$

$= (2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0$ or $\sin\theta - 1 = 0$

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = 1$

when $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\quad\quad$ when $\theta = \frac{\pi}{2}$

Bonus practice test

find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and slope at $\theta = \frac{\pi}{4}$, and concavity at $\theta = \frac{\pi}{4}$

10) $x(\theta) = 2\cos(\theta)$

$y(\theta) = 2\sin(\theta)$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\cos\theta}{-2\sin\theta}$$

$$\boxed{\frac{dy}{dx} = -\frac{\cos\theta}{\sin\theta} \text{ or } -\cot\theta}$$

$\frac{d}{d\theta}[y] = \frac{d}{d\theta}[2\sin\theta]$

$\frac{dy}{d\theta} = 2\cos\theta$

$\frac{d}{d\theta}[x] = \frac{d}{d\theta}[2\cos\theta]$

$= -2\sin\theta$

$\frac{dx}{d\theta} = -2\sin\theta$

Equation of tangent line

$y - y_1 = m_{tan}(x - x_1)$

$y - (\quad) = -1(x - (\quad))$

$y - \sqrt{2} = -x + \sqrt{2}$

$y = -x + 2\sqrt{2}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{d\theta}\left[\frac{dy}{dx}\right]}{\frac{dx}{d\theta}}$$

$$= \frac{\frac{d}{d\theta}[-\cot\theta]}{-2\sin\theta}$$

$$= \frac{\frac{d}{d\theta}[-\cot\theta]}{-2\sin\theta}$$

$$= \frac{-(-\csc^2\theta)}{-2\sin\theta}$$

$$= \frac{\csc^2\theta}{-2\sin\theta}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2}\csc^3\theta$$

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=\frac{\pi}{4}} = \frac{-1}{2\sin^3\frac{\pi}{4}}$$

$=$

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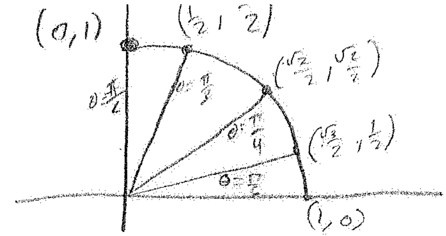
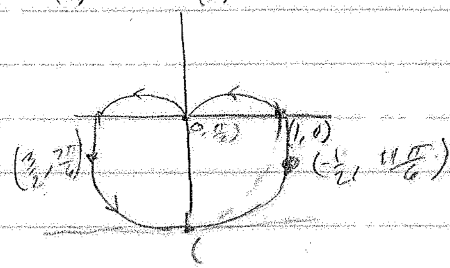


$\boxed{10.4}$ #69

$r(\theta) = 1 - \sin(\theta)$

$r'(\theta) = -\cos(\theta)$



$\frac{dy}{dx}$ = slope of tan. line at a point

$$= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r(\theta)\cdot\cos\theta + r'(\theta)\cdot\sin\theta}{-r(\theta)\cdot\sin\theta + r'(\theta)\cdot\cos\theta}$$

Horizontal line

when $\frac{dy}{d\theta} = 0$, $\frac{dx}{d\theta} \neq 0$

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$\cos\theta = 0$ and $1 - 2\sin\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$1 - 2\sin\frac{\pi}{6} = 0$

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Vertical tangent

when $\frac{dx}{d\theta} = 0$ and $\frac{dy}{d\theta} \neq 0$

$\frac{dx}{d\theta} = -(1 - \sin\theta)\cdot\sin\theta + (-\cos\theta)\cdot\cos\theta$

$= (-1 + \sin\theta)(\sin\theta) + (-\cos^2\theta)$

$= -\sin\theta + \sin^2\theta - \cos^2\theta$

$= -\sin\theta + \sin^2\theta - (1 - \sin^2\theta)$

$= -\sin\theta + \sin^2\theta - 1 + \sin^2\theta$

$= 2\sin^2\theta - \sin\theta - 1$

$= (2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0$ or $\sin\theta - 1 = 0$

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Bonus practice test

find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and slope at $\theta = \frac{\pi}{4}$, and concavity at $\theta = \frac{\pi}{4}$

10) $X(\theta) = 2\cos(\theta)$

$Y(\theta) = 2\sin(\theta)$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\cos\theta}{-2\sin\theta}$$

$$\boxed{\frac{dy}{dx} = -\frac{\cos\theta}{\sin\theta} \text{ or } -\cot\theta}$$

$\frac{d}{d\theta}[y] = \frac{d}{d\theta}[2\sin\theta]$

$\frac{dy}{d\theta} = 2\cos\theta$

$\frac{d}{d\theta}[x] = \frac{d}{d\theta}[2\cos\theta]$

$= -2\sin\theta$

$\frac{dx}{d\theta} = -2\sin\theta$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{d\theta}\left[\frac{dy}{dx}\right]}{\frac{dx}{d\theta}}$$

$$= \frac{\frac{d}{d\theta}[-\cot\theta]}{-2\sin\theta}$$

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$y - \sqrt{2} = -x + \sqrt{2}$

$y = -x + 2\sqrt{2}$

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=\frac{\pi}{4}} = \frac{-1}{2\sin^3\frac{\pi}{4}}$$

$=$